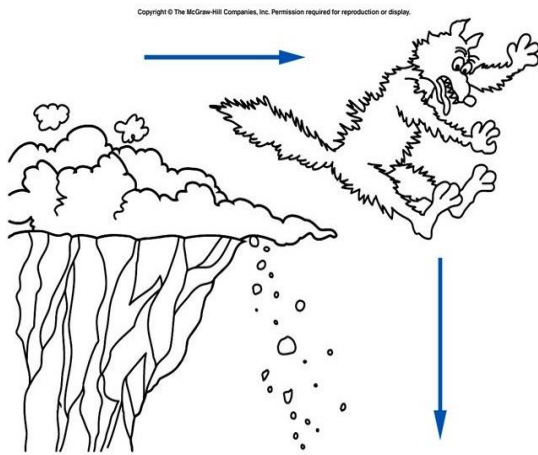


Projectile Motion:

A Cartoon Coyote Falling off a Cliff



We need to consider the horizontal and vertical motions separately projectile motions. Time is the same for both motions.

PHYSICS

1.1 Projectile Motion

ABSTRACT

This study notes have been developed and written to meet the scope and syllabus of all the content of the SACE Stage 2 Physics 2020. The goal of this topic is to enable students not just to recognize concepts, but to work with them in ways that will be useful in final exam.

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Theory

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Subtopic 1.1 Projectile Motion

Students are introduced to the theories and quantitative methods used to describe, determine, and explain projectile motion, both in the absence of air resistance and in media with resistive forces.

Students study projectile motion, through a range of investigations to understand how the principles are applied in the contexts of sports, vehicle designs, and terminal speed.

| Science Understanding | Possible Contexts |
|---|---|
| <p>Uniformly accelerated motion is described in terms of relationships between measurable scalar and vector quantities, including displacement, speed, velocity, and acceleration.</p> <p>Motion under constant acceleration can be described quantitatively using the following formulae:</p> <ul style="list-style-type: none"> • $v = u + at$ • $s = ut + \frac{1}{2}at^2$ • $v^2 = u^2 + 2as$. <p>Projectile motion can be analyzed quantitatively by treating the horizontal and vertical components of the motion independently.</p> <ul style="list-style-type: none"> • Construct, identify, and label displacement, velocity, and acceleration vectors. • Use vector addition and subtraction to calculate net vector quantities. • Resolve velocity into vertical and horizontal components, using $v_H = v \cos \theta$ and $v_v = v \sin \theta$ for the horizontal and vertical components respectively. • Determine the velocity at any point, using trigonometric calculations or a scale diagram. | <p><i>Explanation of the difference between scalar and vector quantities and methods of measurement of these quantities is covered in Stage 1, Topic 1: Linear Motion and Forces.</i></p> <p><i>This uses the concept of acceleration developed in Stage 1, Subtopic 1.1: Motion under Constant Acceleration.</i></p> <p>Use trigonometric calculations and scale diagrams to determine quantities, using vector addition and subtraction.</p> <p>Given a diagram showing the path of a projectile, draw vectors to show the forces acting on the projectile, as well as the acceleration and velocity vectors.</p> <hr/> <p>Use a projectile launcher to investigate the effect of launch angle or launch height on range.</p> |
| <p>An object experiences a <u>constant gravitational force</u> near the surface of the Earth, which causes it to undergo <u>uniform acceleration</u>.</p> <ul style="list-style-type: none"> • Explain that the acceleration of a projectile is always downwards and independent of its mass. • Explain that, in the absence of air resistance, the horizontal component of the velocity is constant. | <p>Demonstrate the independence of acceleration due to gravity on mass using NASA footage of dropping a hammer and feather on the Moon:</p> <p>https://www.youtube.com/watch?v=5C5_dOEyAfk</p> |

| Science Understanding | Possible Contexts |
|---|---|
| <p>The motion formulae are used to calculate measurable quantities for objects undergoing projectile motion.</p> <ul style="list-style-type: none"> • Calculate the <u>time of flight</u> when a projectile is launched horizontally. • Calculate the time of flight and the <u>maximum height</u> for a projectile when the launch height is the same as the landing height. • Calculate the <u>horizontal range</u> of a projectile. • Explain qualitatively that the <u>maximum range</u> occurs at a launch angle of 45° for projectiles that land at the same height from which they were launched. • Describe the relationship between launch angles that result in the same range. • Describe and explain the effect of launch height, speed, and angle on the time of flight and the maximum range of a projectile. • Analyze multi-image representations of projectile paths. | <p>Investigate the ‘monkey and the hunter’ problem both quantitatively and qualitatively.</p> <p>Use video footage to analyse projectile motion in a variety of contexts.</p> <p>Analyze the constant horizontal component of the velocity of the projectile qualitatively and quantitatively, using various recording technologies.</p> <p>Model and demonstrate that the maximum range occurs at a launch angle other than 45° when the launch height is different to the landing height.</p> <p>In terms of projectile motion, analyse footage of students undertaking a sport like shot put.</p> <p>Use concepts from projectile motion to analyse sporting activities such as aerial skiing, golf, javelin, shot put, and various ball sports.</p> |
| <p>When a body moves through a medium such as air, the body experiences a <u>drag force</u> that opposes the motion of the body.</p> <ul style="list-style-type: none"> • Explain the effects of speed, cross-sectional area, and density of the medium on the drag force on a moving body. • Explain that <u>terminal velocity</u> occurs when the magnitude of the drag force results in zero net force on the moving body. • Describe situations such as skydiving and the maximum speed of racing cars where terminal velocity is achieved. • Describe and explain the effects of air resistance on the vertical and horizontal components of the velocity, maximum height, and range of a projectile. • Describe and explain the effects of air resistance on the time for a projectile to reach the maximum height or to fall from the maximum height. | <p>Determine the <u>terminal velocity</u> of a spherical object by dropping it into a viscous liquid.</p> <p>Determine the drag coefficients by dropping coffee filters or cupcake holders. By manipulating the mass and recording the time taken to reach the ground, use the air resistance formula to calculate the drag coefficient.</p> <p>Discuss the conclusions of experiments comparing swimming in syrup with swimming in water:</p> <p>http://www.nature.com/news/2004/040920/full/news040920-2.html</p> <p>Explore examples of the way that scientists have been able to develop solutions affecting aerodynamics (such as shape, texture, and spin) of different objects like balls, planes, and cars.</p> |

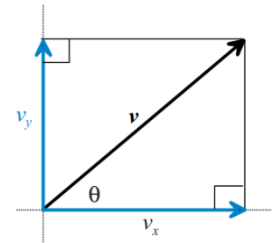
1 Components of Vectors

1.1 Resolving a Vector into its Components

All vectors can be resolved into two mutually perpendicular components.

Most commonly, we find the components of vectors in the horizontal and vertical directions. But this does not have to be so.

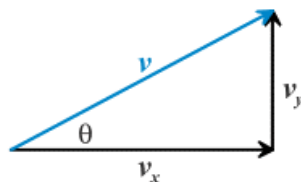
The components of the vector can be found in any pair of directions – the only limiting condition is that the two directions must be perpendicular to each other.



Hence a vector is equal to the sum of its components

1.2 Finding a Vector from its Components

If the components of a two-dimensional vector are known, then the Magnitude of the vector can be found by application of Pythagoras' theorem.



$$v^2 = v_x^2 + v_y^2$$

$$v = \sqrt{v_x^2 + v_y^2}$$

The Direction of the vector can be found by the application of right-angled triangle rule.

$$\tan \theta = \frac{v_y}{v_x}$$

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right)$$

Note

- ✚ Motions in two dimensions can be analysed as the vector sum of two mutually perpendicular, one-dimensional motions.

We use the components of force, acceleration, velocity and displacement in these directions.

The components can be taken in any two, mutually perpendicular, directions

- ✚ No vector can have any effect, action or component in a direction perpendicular to itself.

Because of this we can treat the horizontal and vertical components of the parameters of the

motion as completely independent of each other. Thus, we can treat two-dimensional motions as two independent motions in the horizontal and vertical directions. That is, two separate, independent, one-dimensional motions.

✚ All vectors can be resolved into their horizontal and vertical components.

✚ Given its components, the original vector can be found.

1.3 Motion Parameters

In Motion study, the following vector parameters are used.

Displacement 'S'

The displacement 'S' of a body is equal to the vector sum of its Easterly and Northerly components.

$$S = ut + \frac{1}{2}at^2$$

$$S = vt - \frac{1}{2}at^2$$

$$S = \frac{1}{2}(v + u)t$$

Velocity 'v'

The velocity 'v' of a body is equal to the vector sum of its Easterly and Northerly components

$$v = u + at$$

$$v^2 = u^2 + 2aS$$

Acceleration 'a'

Acceleration is constant and always directed vertically ↓ downwards.

Force 'F'

The Force 'F' of a body is equal to the vector sum of its Easterly and Northerly components

$$F = ma$$

2 Projectile Motion

A projectile is any object that has been projected (or launched or thrown or fired) at some angle into the air, near the surface of the earth. The subsequent motion of this object is a parabola, by neglecting the air resistance.

The analysis of this two-dimensional projectile motion relies on the following techniques.

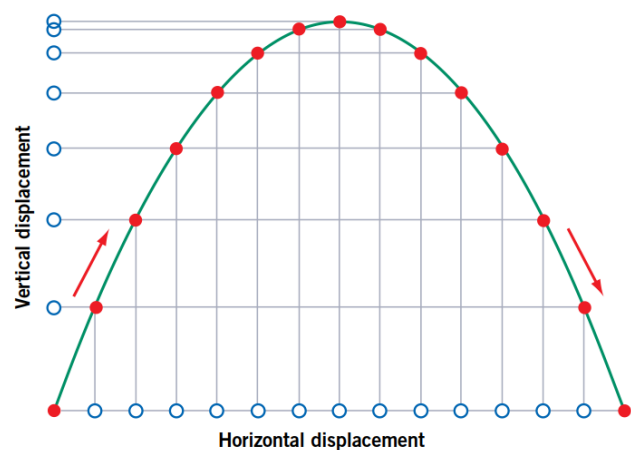
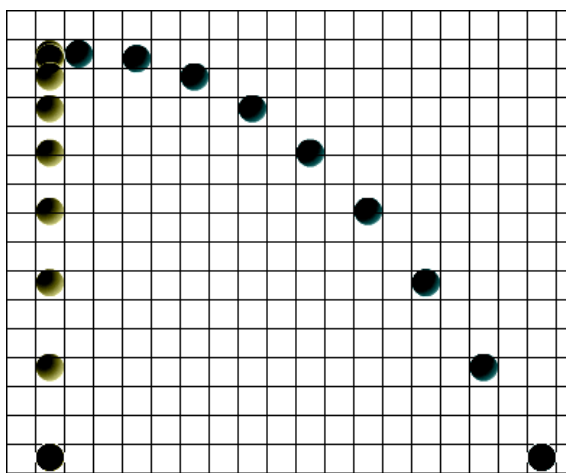
- ✚ Resolve the motion into two perpendicular one-dimensional motions.
- ✚ Treating these two components of the motion independent of each other.

2.1 Vertical and Horizontal Components of Velocity

A projectile's velocity is a vector and can be resolved into two components (Horizontal v_x and Vertical v_y). These components are vectors at right angles to each other.

A multi-image photograph can be used to demonstrate the behaviour of the components.

In a multi-image photograph, the time difference between images is constant.



Since the projectile moves the same horizontal distance every time, its horizontal component of velocity must be constant. $s_x = u_x t$.

Therefore, to draw the images of the projectile with equal horizontal spacing between them.

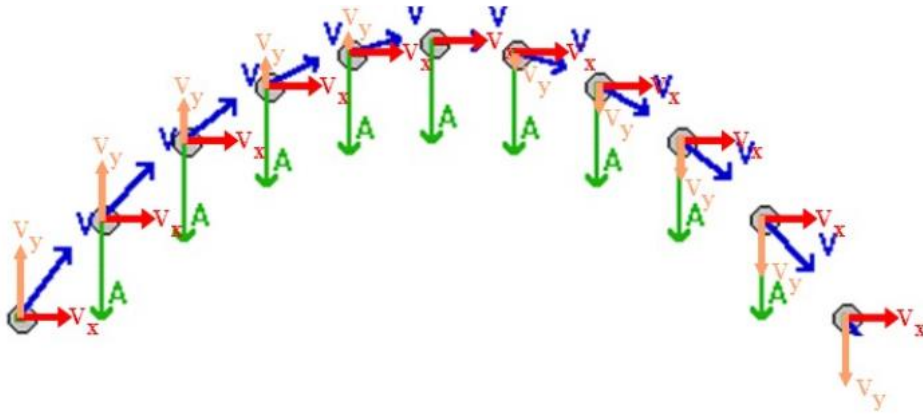
Both objects have zero initial vertical component of velocity to begin with. They fall to the ground in the same amount of time, which shows:

- ✚ Horizontal motion and vertical motion are independent of each other
- ✚ Projectiles have the same acceleration as a vertically free-falling object

Note

- ✚ The path of the projectile is parabola.
- ✚ A projectile is an object moving through the air in a curved trajectory with no propulsion system.
- ✚ The horizontal component of the velocity remains constant throughout the flight.
- ✚ The vertical component of the velocity decreases as the projectile rises; and increases as the projectile falls:
- ✚ At the topmost point the vertical component of the velocity is zero.
- ✚ The direction of motion of velocity is always at a tangent to the parabola at all times. The magnitude and direction of velocity changes with time.
- ✚ Acceleration is constant and always directed vertically ↓ downwards.
- ✚ The force of air resistance always acts opposite to the direction of motion of velocity.
- ✚ A projectile is an object that only moves under the influence of gravity
- ✚ The vertical and horizontal components of motion are independent
- ✚ Projectile motion is motion with a constant horizontal velocity combined with a constant vertical acceleration.
- ✚ The horizontal and vertical motions of a projectile are independent of each other except they have a common time.
- ✚ Projectile motion problems can be solved by applying the constant velocity equation for the horizontal component of the motion and the constant acceleration equations for the vertical component of the motion.
- ✚ Pythagoras' theorem can be used to determine the actual speed of the projectile at any point.

3 Velocity



- ✚ The velocity vector, v , is always at a tangent to the parabola.
- ✚ The direction of the velocity and the magnitude of the velocity changes with time.
- ✚ The horizontal component of the velocity v_x remains constant throughout the motion. Therefore, when sketching, the horizontal components are all in the same length.
- ✚ The vertical component of the velocity v_y changes with time, decreasing in magnitude while the projectile is rising.
- ✚ Once the projectile passes the highest point of its flight, the vertical component reverses direction, and increases in magnitude in the downward direction.
- ✚ At the highest point of its flight the velocity of the projectile is in the horizontal direction. The vertical component of the velocity here, is zero ($v_y = 0$). The actual velocity at this point is equal to horizontal component velocity only. ($v = v_x$).
- ✚ The vertical and horizontal components v_x and v_y of velocity at any point are known, the actual velocity v at that point can be found by vector addition of these components, i.e.
$$v = v_x + v_y$$
- ✚ The acceleration is constant and always directed vertically downward.

3.1 Resolution of Initial Velocity into Components

In case of horizontal projection, the initial vertical component of velocity ($u_y = 0$) is zero and the horizontal component is equal to the initial velocity. i.e. ($u_x = u$).

In all other cases, where the body is initially projected with initial velocity u at angle θ to the horizontal, the following components are applicable. In which the horizontal component of velocity remains constant throughout the motion.

- ✚ Horizontal component of initial velocity ($u_x = u \cos \theta$)
- ✚ Vertical component of initial velocity ($u_y = u \sin \theta$)

3.2 Determination of Final Velocity

The final velocity of the body can be found by the vector addition of the horizontal and vertical components of final velocity. Note that the horizontal component of final velocity is nothing but the horizontal component of the initial velocity.

$$(v_x = u_x)$$

The vertical component of final velocity can be found by using Newton's first law of motion.

$$(v_y = u_y + a_y t).$$

By using Pythagoras theorem, the final velocity v^2 is

$$(v^2 = v_x^2 + v_y^2)$$

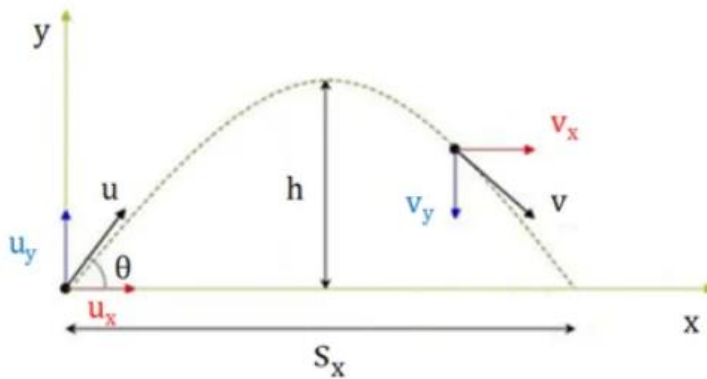
The angle will be by trigonometry

$$\tan \theta = \frac{v_y}{v_x}$$

$$\theta = \tan^{-1} \frac{v_y}{v_x}$$

Note:

The velocity of a projectile body at any time is at a tangent to the parabolic path traced out. The vertical and horizontal components of velocity at any time, when we add them vectorially, will give the instantaneous velocity at that point.



$$u_x = u \cos \theta$$

$$u_y = u \sin \theta$$

$$u_x = v_x \text{ (horizontal component)}$$

$$v_y = 0 \text{ (at the max height 'h')}$$

$$S_x = u_x * t \text{ (as normal equation)}$$

$$S_y = h \text{ (where } v_y = 0 \text{ and } v_x = u_x)$$

4 Time of Flight

In situations where the initial height and the final height are same, (uni-level), the time of flight can be found by calculating the time taken for the body to reach its maximum height and the doubling it.

In general cases, the time flight of the projectile can be found by applying the equation

$$S_y = u_y t + \frac{1}{2} a_y t^2 \text{ in the vertical direction.}$$

The parameter S_y represents the displacement of the body in vertical position.

5 Maximum Height

The maximum height reached is the maximum vertical displacement from the launch position plus the launch height. The maximum vertical displacement from the initial launch position is achieved at the peak of the trajectory, when the vertical component of the velocity $v_y = 0$. The maximum vertical displacement above the launch position is most easily calculated as follows.

$$v_y^2 - u_y^2 = 2a_y S_y$$

$$0 - u_y^2 = -2g S_{y \max}$$

$$S_{y \max} = \frac{u_y^2}{2g}$$

$$S_{y \max} = \frac{u^2 \sin^2 \theta}{2g}$$

6 Range

The range of a projectile can be determined by multiplying the horizontal velocity by the time of flights. The horizontal velocity is constant. Therefore, the horizontal distance (Range) travelled is just horizontal velocity times time.

$$\text{Range } S_x = u_x t = u \cos \theta t$$

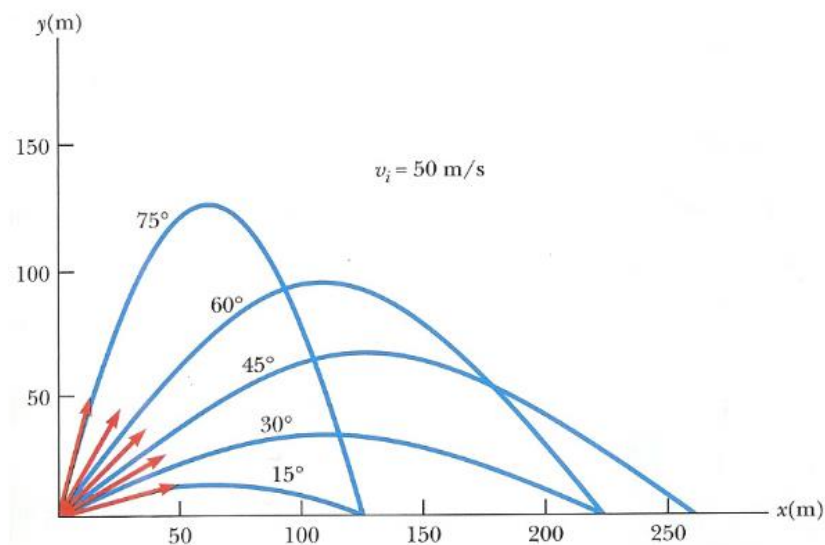
6.1 Relationship between Range & Launch Angle

The following conclusions can be made in the case of projection of body in uni-level scenario.

Conclusion 1: For a projectile of given speed, the Range will be maximum if the launch angle is 45° above the horizontal

Conclusion 2: For two projectiles of given speed, the Range will be the same if the two launch angles are symmetrical about 45° or if the two launch angles add up to 90° .

Proof



Consider a projectile launched with speed u and launch angle θ

Vertical acceleration: $a_y = -g = -9.8 \text{ ms}^{-2}$

Horizontal and vertical components of initial velocity are given by

$$u_x = u \cos \theta$$

$$u_y = u \sin \theta$$

Time of flight calculation

$$S_y = u_y t - \frac{1}{2} a_y t^2$$

$$0 = (u \sin \theta) t - \frac{1}{2} g t^2$$

$$0 = t(u \sin \theta - \frac{1}{2} g t)$$

$$t = 0 \text{ or } t = \frac{2u \sin \theta}{g}$$

$$\boxed{t = \frac{2u \sin \theta}{g}}$$

Range calculation

$$\text{Range } S_x = u_x t$$

$$S_x = u \cos \theta \cdot \frac{2u \sin \theta}{g}$$

$$S_x = \frac{u^2 2 \sin \theta \cos \theta}{g}$$

$$\boxed{S_x = \frac{u^2 \sin 2\theta}{g}}$$

Therefore, range will be maximum when $\sin 2\theta$ is maximum

$$\text{When } \sin 2\theta = 1 \rightarrow 2\theta = 90^\circ \rightarrow \theta = 45^\circ$$

7 Five Scenarios of Projectile Motions

7.1 Horizontal Projection

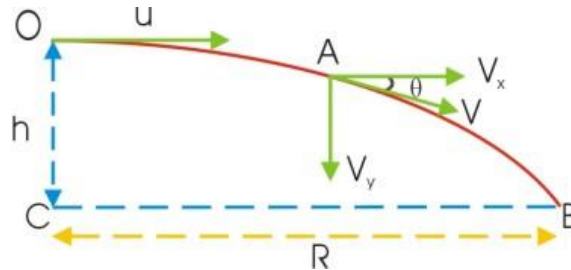
A body is projected horizontally with speed u from some height h and finishes its flight at some point below its initial height.

The initial horizontal velocity $u_x = u$

The initial vertical velocity $u_y = 0$

The acceleration $a_y = 9.8 \downarrow$

The final vertical displacement $s_y = h$



Example

- ✚ A ball rolling off a table
- ✚ Supplies being dropped from an aeroplane

7.2 Downward Projection

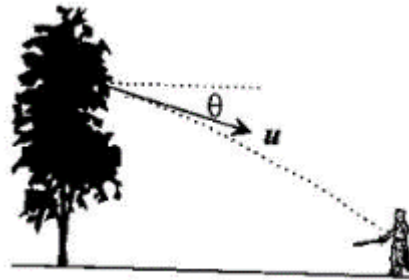
A body is projected with a speed u at an angle θ below horizontal and finishes its flight at a level h meters below its initial height.

The initial horizontal velocity $u_x = u \cos \theta$

The initial vertical velocity $u_y = u \sin \theta$

The acceleration $a_y = 9.8 \downarrow$

The final displacement $s_y = h$



Example

- ✚ Throwing a ball off a roof
- ✚ Robin Hood shooting an arrow from his position in a tree.

7.3 Uni-level angular Projection

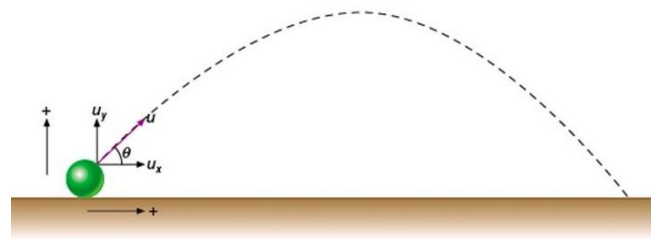
A body is projected with a speed u at an angle θ above the horizontal and finishes its flight at the same level.

The initial horizontal velocity $u_x = u \cos \theta$

The initial vertical velocity $u_y = u \sin \theta$

The acceleration $a_y = -9.8 \uparrow$

The final vertical displacement $s_y = 0$



Example

- ✚ An athlete competing in the high jump

7.4 Bi-level angular Projection

Case 1

A body is projected with a speed u at an angle θ above horizontal. The launch point is some distance h above ground level. The projectile finishes its flight at a level below its initial height.

The initial horizontal velocity $u_x = u \cos \theta$

The initial vertical velocity $u_y = u \sin \theta$

The acceleration $a_y = -9.8 \uparrow$

The final displacement $s_y = -h$



Example

- ✚ An athlete putting his shot
- ✚ A driver jumping off a springboard

Case 2

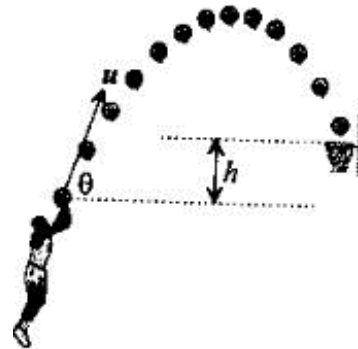
A body is projected with a speed u at an angle θ above horizontal and finishes its flight at a level h above its initial height.

The initial horizontal velocity $u_x = u \cos \theta$

The initial vertical velocity $u_y = u \sin \theta$

The acceleration $a_y = -9.8 \uparrow$

The final displacement $s_y = h$



Example

- ✚ A basketball shooter from free flow line

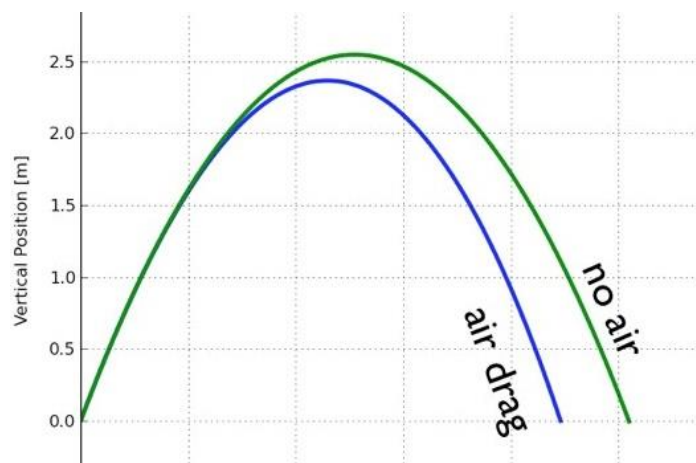
8 Air Resistance

Air resistance acts on all objects that move through air. This is because a body must push air out of the way to move through it. This requires a force, and when the body exerts this force it does work, thus losing energy. Air resistance is acting in a direction opposite to the direction of motion (i.e. velocity) of the body. In other words, it is a frictional force.

The motion (i.e. velocity) of the projectile is constantly changing in magnitude and direction. In same way the air resisting force does. Just as the velocity of the projectile has components in the horizontal and vertical directions, so too will the air resisting force have force components in these directions.

The net effect of these force components will be to reduce the magnitudes of the horizontal and vertical components of the velocity of the projectile.

To an observer, the effect of this will be much more noticeable on the horizontal velocity of the body than on the vertical velocity.



Once the projectile has passed the highest point of its flight, the vertical component of velocity is continually increasing in magnitude because of the force of gravity in downward direction. But Air resistance will have a vertical component in upward direction. (i.e. opposite to gravitational field). The net vertical force on the body is the vector sum of these two opposing forces. Thus, the effect of this air resistance is to reduce the downward force on the projectile, and hence to reduce the downward acceleration also. To an observer, the effect of this reduction would probably not be immediately noticeable in most cases.

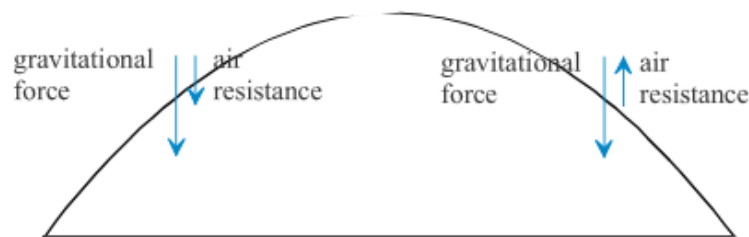
The horizontal velocity is constant throughout the projectile path. The effect of air resistance is to continually keep reducing the horizontal component of the velocity of the projectile.

Note

- ✚ On the upward path, air resistance has a vertical component F_y acting \downarrow downward. Thus, gravity and air resistance both act together to retard the vertical motion of projectile.
- ✚ On the downward path, air resistance has a vertical component F_y acting \uparrow upward. Thus, gravity and air resistance act in opposite direction. Therefore, it takes more time to fall to ground from its highest point.
- ✚ The horizontal component of air resistance acts opposite to the horizontal component of the velocity throughout the motion. So, the range is consequently reduced.

8.1 Effect of Air Resistance on Time of Flight

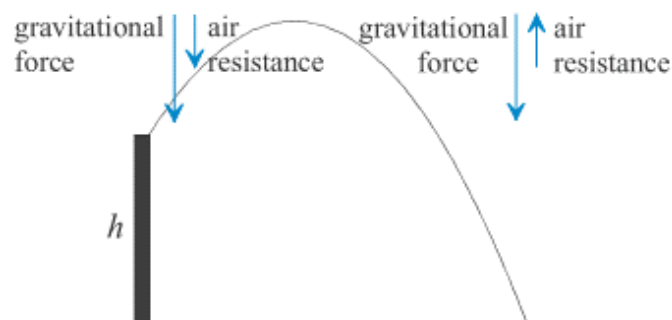
Uni-level Projection



In the first half of the motion, when the body is on the way up, the force of gravity and the air resistance are both acting in the same \downarrow downward direction. This increases the net force, retarding the motion of the body. Thus, the time taken to reach the highest point of motion is reduced and the body will not rise as far.

In the latter half of the motion, the force of gravity and the air resistance are in opposite directions. This results in a net accelerating force downwards, which is less than gravitational force. Therefore, it will take longer time for the body to fall the same distance. (But the distance that it must fall is less.)

Bi-level Projection



In the case of bi-level projection, where the body is launched from a height h above ground level, the same considerations apply as above. However, there is a much longer downward path. The net

downward force is reduced due to the opposing force of air resistance. Therefore, the downward velocity is less and thus the time of flight is increased.

8.2 Effect of Air Resistance on Range

Because the horizontal component of velocity is continually decreasing because of air resistance, the range of the projectile is decreased.

8.3 The Significance of Air Resistance in Sports

The air resistance or aerodynamic drag on a body is a force, the magnitude of which is given by

$$R = \frac{1}{2} D \rho A v^2$$

where

R is Resistive force (N)

ρ is density of air

A is cross sectional area of projectile (m^2)

v is the speed of the projectile (ms^{-1})

D is Drag coefficient (dimension less constant) (Shape and surface texture)

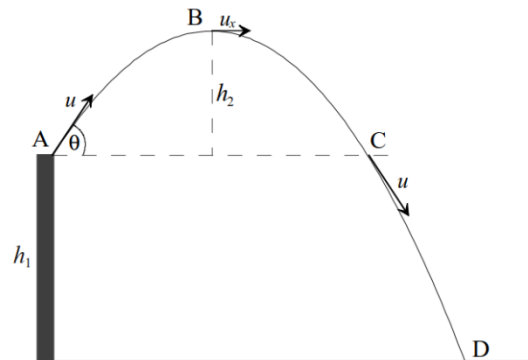
Note

- ✚ Air resistance is directly proportional to the square of the speed of the body. Therefore, as speed increases, the air resistance would become much more significant.
- ✚ Air resistance is directly proportional to the cross-sectional area of the body. Thus, bodies with large cross-sectional area (soccer ball) experience more air resistance than smaller one (golf ball).
- ✚ Air resistance is depending on the shape and surface texture of the body. The drag coefficient has a value of about 0.5 for sphere and as high as 2 for irregular shape of body.
- ✚ Air resistance is proportional to air density. Density of air depends up on Air pressure, Air temperature and Humidity.

9 Energy Consideration

The Law of Conservation of Energy states: “Energy cannot be created or destroyed but may be converted from one form to another”.

At all points of the projectile’s path the total energy must remain constant, because the body is moving in the Earth’s gravitational field, the total energy of the body consists of the sum of the kinetic energy of the body and its gravitational potential energy.



Consider a body being launched from a height h_1 above the Earth’s surface with initial speed u , at an angle θ above the horizontal.

At point A,

$$E_T = mgh_1 + \frac{1}{2}mu^2$$

As the body climbs, kinetic energy is being converted to gravitational potential energy, which increases. The loss in kinetic energy is at all points equal to the gain in gravitational potential energy.

At point B, the highest point

$$E_T = mg(h_1 + h_2) + \frac{1}{2}mu_x^2$$

The body has gained gravitational potential energy equal to $PE = mgh_2$ and has lost that amount of kinetic energy

At point C,

when the body has returned to its initial height above the Earth’s surface, its total energy consists of the same amounts of gravitational potential energy and kinetic energy as at the beginning of the motion.

$$E_T = mgh_1 + \frac{1}{2}mu^2$$

At point D,

Just before the body hits the ground, all its gravitational potential energy has been converted to kinetic energy.

$$E_T = 0 + \frac{1}{2}mv^2$$

10 Solving Problems

General Points

The strategy is to consider the horizontal and vertical components of the motion separately and use the equations of motion in each direction.

- ✚ The vertical acceleration is always -9.8 ms^{-2} (due to gravity)
- ✚ The projectile will continue to move upwards until the vertical component of its velocity is zero
- ✚ The horizontal acceleration is always zero
- ✚ The horizontal velocity is constant

Strategies

There are various quantities that you are commonly asked to find. These are the strategies to find them:

| | |
|--|---|
| Time to the highest point | use $v_y = u_y + a_y * t$ (vertically), [with $v_y = 0$, & $a_y = -9.8 \text{ ms}^{-2}$] |
| Greatest vertical height | use $2 a_y s_y = v_y^2 - u_y^2$ (vertically), [with $v_y = 0$ & $a_y = -9.8 \text{ ms}^{-2}$], or use $s_y = u_y t + \frac{1}{2} a_y t^2$ if the time is known |
| Time taken for it to reach a particular height | use $s_y = u_y t + \frac{1}{2} a_y t^2$ (vertically) (if it is returning to the same height at which it started – [e.g. returning to the ground - take $s_y = 0$]) |
| Total horizontal distance travelled | find the time taken for it to finish its journey (as above), then use $s_x = u_x * t$ horizontally |

11 Types of question

There are three common situations, when an object is:

1. Thrown vertically upwards or downwards
2. Projected with a horizontal velocity
3. Projected at an angle to the horizontal

The first one is not conventionally called a projectile, but can be examined, and understanding it helps with the last two.

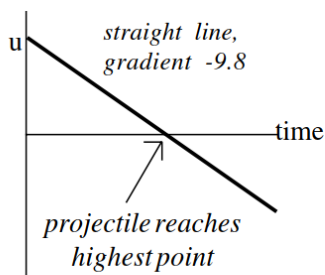
11.1 Object thrown vertically

- ✚ No horizontal component - consider vertical motion only.
- ✚ Start velocity: u has +ve value if thrown upwards, -ve value if downwards
- ✚ There is vertical acceleration $g = -9.8 \text{ ms}^{-2}$.
- ✚ If the object is thrown upwards, it continues to rise until $v = 0$

If the particle is thrown upwards, the graphs are as follows:

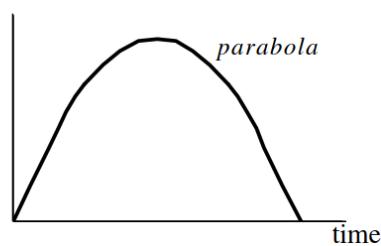
velocity-time graph

vertical
velocity



displacement-time graph

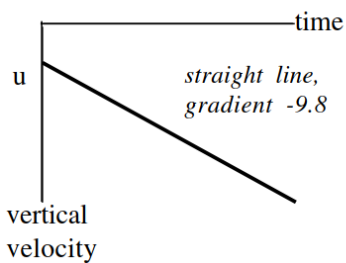
vertical
displacement



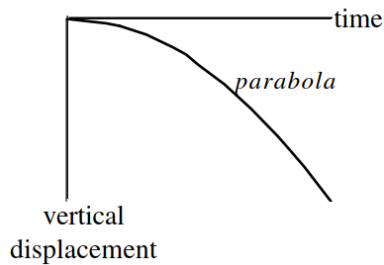
Note: This assumes the projectile returns to the same vertical level from which it started.

If the particle is thrown downwards, the graphs are as follows:

velocity-time graph



displacement-time graph

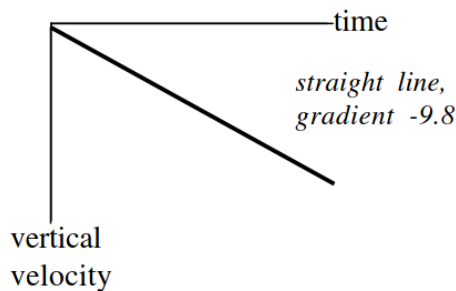


11.2 Object projected horizontally

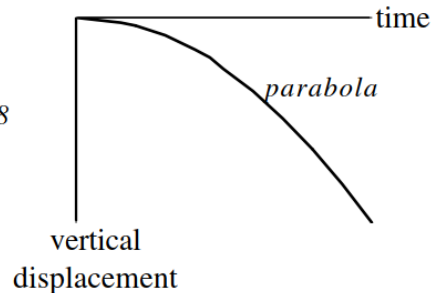
- ✚ Start velocity has no vertical component, only a horizontal one
- ✚ Acceleration is $g = -9.8 \text{ ms}^{-2}$ vertically, no horizontal acceleration
- ✚ Object continues to move until it falls to the ground.
- ✚ All objects launched horizontally or dropped from the same height will hit the ground at the same time. Their vertical component of velocity is the same at all times.
- ✚ If the question asks for a velocity at a certain time or point with no direction mentioned, then it will mean resultant velocity. You should give both magnitude and direction (angle) in your answer.

Vertical Motion

velocity-time graph

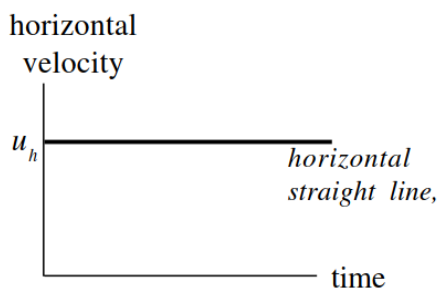


displacement-time graph

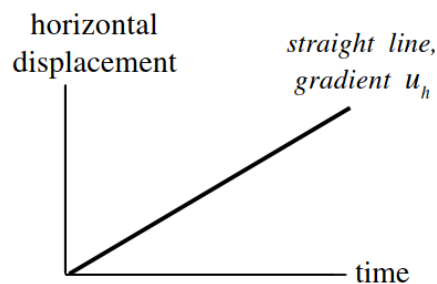


Horizontal Motion

velocity-time graph



displacement-time graph



11.3 Launched at an angle to the horizontal

- ✚ Start velocity has both horizontal and vertical components
- ✚ Acceleration is $g = -9.8 \text{ ms}^{-2}$ vertically, no horizontal acceleration
- ✚ Object continues to move until it falls to the ground.
- ✚ In many cases, the object returns to the same height from which it started, but this is not always the case (e.g. object thrown upwards from a cliff)

